

Subluminal OPERA Neutrinos

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Abstract

The OPERA collaboration has announced to have observed superluminal neutrinos with a mean energy 17.5 GeV, but afterward the superluminal interpretation of the OPERA results has been refuted theoretically by Cherenkov-like radiation and pion decay. In a recent work, we have proposed a kinematical resolution to this problem. A key idea in our resolution is that the OPERA neutrinos are not superluminal but subluminal since they travel faster than the observed speed of light in vacuum on the earth while they do slower than the true speed of light in vacuum determining the causal structure of events. In this article, we dwell upon our ideas and present some concrete models, which realize our ideas, based on spin 0, 1 and 2 bosonic fields. We also discuss that the principle of invariant speed of light in special relativity can be replaced with the principle of a universal limiting speed.

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1 Introduction

The OPERA collaboration has recently claimed that it has observed a superluminal speed of muon neutrinos [1]. The CNGS beam of neutrinos with a mean energy 17.5 GeV ranging up to 50 GeV travels along the baseline distance 730km from CERN to the Gran Sasso laboratory. The OPERA has found that the neutrinos arrived earlier than expected from the speed of light by about 60 nano-seconds. (In the recent second experiment using a shorter neutrino beam, the time is about 62 nano-seconds faster than the speed of light which is within errors of the first experiment.)

An important theoretical challenge is to reconcile the OPERA superluminal neutrinos with the subluminal neutrinos traveled from SN1987A [2]. Average energy of the OPERA neutrinos is a thousand times larger than that of the SN1987A neutrinos. This observation prompts some people to hit on an idea that only high-energy neutrinos might be superluminal whereas low-energy ones are subluminal. The simplest possible approach realizing this idea is to change the conventional dispersion relation of a neutrino by adding the Lorentz-invariant and/or the Lorentz-noninvariant terms, thereby making it possible to change the velocity of the neutrino in an energy-dependent way in order to agree with both the experiments. However, although this resolution might be a logical possibility, we think it quite unnatural since both the OPERA superluminal neutrinos with about 17 GeV and the SN1987A subluminal ones with about 20 MeV are already in the ultra-high energy region compared to its characteristic energy scale, which is equal to mass of neutrinos, $m(\nu_e) \leq 2.5eV$ for an electric neutrino and $m(\nu_\mu) \leq 170keV$ for a muon neutrino [3].

The other challenges to the OPERA experimental results are the bremsstrahlung effect [4] and pion decay [5, 6, 7]. The bremsstrahlung effect (or sometimes called Cherenkov-like radiation or Cohen-Glashow effect) of superluminal neutrinos is a characteristic feature of *superluminal* neutrinos with weak interaction such that even if neutrinos do not carry electric charges, the excessive kinetic energy of the superluminal motion is converted to energy for creating a pair of electron and positron. After some calculations, it turns out that on the way from CERN to Gran Sasso, the very effect of superluminal propagation of neutrinos would have caused some distortions in the beam of neutrinos owing to the bremsstrahlung effect and severely depleted the higher-energy neutrinos, thereby making it impossible to observe neutrinos with more than 12.5 GeV energy. This theoretical result is obviously against the OPERA results where a lot of high-energy neutrinos above 12.5 GeV are observed [1].

More recently, stimulated with the above theoretical observation, ICARUS group has analyzed their data and found that the neutrino energy distribution of the ICARUS events in IAr does not have such a distorted energy distribution of beam from CERN. Thus, the ICARUS group also refutes a superluminal interpretation of the OPERA results on the basis of the Cohen and Glashow prediction for a weak current analog to Cherenkov radiation [8].

In this article, let us suppose that the OPERA results are correct even if further experimental scrutiny is surely needed. It is then true that a confirmation of the superluminal neutrino motion might require a radical reconsideration of fundamental principles behind particle physics. A successful theory which explains the OPERA results quantitatively as well

as qualitatively should be consistent with very restrictive bounds on the violation of Lorentz invariance in the sector of charged particles, with the absence of abnormal dispersion of the neutrino signal from SN1987A, and with the absence of intensive neutrino decays which are characteristic features for many models with derivations from the relativistic invariance.

However, since special relativity has passed many of nontrivial both experimental and theoretical tests thus far and has a firm foundation, we believe that special relativity cannot be ruled out by such a single experiment of the speed of neutrinos. We therefore wish to conjecture that if the OPERA report is correct, it might suggest that the concept of the velocity of light in vacuum must be modified to some degree without changing the essential contents of special relativity.²

We shall postulate existence of new bosonic degrees of freedom with spins 0, 1 and 2, and then assume that these new fields are sourced by the earth, in particular, its energy-momentum tensor or electro-magnetic current and create a classical background. In contrast to previous models, it is assumed that in our models only photons couple to the classical background and consequently they propagate on the background via an effective metric.

This article is organized as follows: In the next section, we wish to explain that the principle of invariant light speed in special relativity can be replaced with the principle of a universal limiting speed. This limiting speed is nothing but the speed which appears in various formulae in special relativity. In Section 3, we will present three models which have two speeds of light because of interactions between the gauge field and bosonic fields with spins 0, 1 and 2, which are generalizations of our previous model [13]. Section 4 is devoted to discussion.

2 Review of special relativity and our resolution to Cohen-Glashow effect

In this section, we review special relativity [14], in particular, the principle of invariant speed of light and present our resolution to Cohen-Glashow effect of superluminal neutrinos.

The special theory of relativity by Einstein, which is often called *special relativity*, has become a commonplace in physics, as taken for granted as Newton's laws of classical mechanics and Maxwell equations of electrodynamics. It is well known that Einstein's special relativity is based on two fundamental principles, those are, the principle of special relativity and the principle of invariant speed of light.

The former principle says that all physical equations must be invariant under Lorentz transformations and it is a universal principle which holds for every physical phenomenon. On the other hand, the latter principle, which implies that the speed of light is finite and independent of the motion of its source, supposes existence of light from the outset. Here note

²A similar idea has been put forward by Nakanishi [9]. See also related works [10, 11, 12].

that light is a kind of electromagnetic waves whose existence is guaranteed by electromagnetics. In this context, it seems to be a bit strange to accept the principle of invariant light speed as one of the fundamental principles in special relativity since this principle attaches too much importance to electrodynamics. Special relativity as well as general relativity are theories of space and time so they should be defined prior to other branches of physics such as electrodynamics.

It is therefore natural to imagine that we might be able to construct special relativity without relying on the existence of light coming from electrodynamics. Indeed, it was known quite some time ago that we can make special relativity in a such way that it is not based on the principle of invariant light speed as one of the fundamental principles [15, 16, 17, 18]. In such an approach, the principle of invariant speed of light is replaced with the principle of a universal limiting speed, which means that in every inertial frame, there is a finite universal limiting speed C for all physical entities. In passing we remark that the existence of a universal limiting speed could be rephrased that there is a upper limit of speed in the propagation of information.

Now we wish to show explicitly that both the principle of special relativity and that of a universal limiting speed produce the Lorentz transformation and the composition law of velocities in special relativity. To do that, let I and I' be two inertial, non-accelerating reference frames in such a way that I' moves in the direction of the x axis with a relative velocity v compared to I . (For simplicity we omit y and z coordinates.) Then, it is easy to see that the principle of special relativity together with isotropy and homogeneity of the Minkowski space-time leads to the most general transformation between the coordinates (t, x) in I and (t', x') in I'

$$\begin{aligned} t' &= A(v^2)t - B(v^2)vx, \\ x' &= D(v^2)(x - vt), \end{aligned} \tag{1}$$

and its inverse transformation

$$\begin{aligned} t &= A(v^2)t' + B(v^2)vx', \\ x &= D(v^2)(x' + vt'), \end{aligned} \tag{2}$$

where A , B and C are functions of only v^2 , and note that the common factor D is needed from the definition of inertial frames in the relative motion.

With this most general transformation, the consistency condition between (1) and (2) yields relations

$$A = D, \quad (A - v^2B)D = 1. \tag{3}$$

If a physical object has a velocity V' in the reference frame I' , the corresponding velocity V in the reference frame I reads

$$V \equiv \frac{dx}{dt} = \frac{\frac{dx}{dt'}}{\frac{dt}{dt'}} = \frac{V' + v}{1 + vV'\frac{B}{A}}, \tag{4}$$

where Eq. (3) is used. Then, the principle of a universal limiting speed C allows us to choose

$$\frac{B}{A} = \frac{1}{C^2}. \quad (5)$$

As a result, we arrive at the Lorentz transformation

$$\begin{aligned} t' &= \frac{t - \frac{v}{C^2}x}{\sqrt{1 - (\frac{v}{C})^2}}, \\ x' &= \frac{x - vt}{\sqrt{1 - (\frac{v}{C})^2}}, \end{aligned} \quad (6)$$

and the composition law of velocities

$$V = \frac{V' + v}{1 + \frac{vV'}{C^2}}. \quad (7)$$

Moreover, it has been shown by Terletsii [19] that by considering three inertial reference frames $I(t, x)$, $I'(t', x')$ and $I''(t'', x'')$, and the group property of the two transformations $(t, x) \rightarrow (t', x') \rightarrow (t'', x'')$ and $(t, x) \rightarrow (t'', x'')$ directly, C is a universal constant with the dimension of a speed. In addition to it, one can show within the framework of this approach that massive particles cannot move with the velocity of the universal limiting speed C whereas massless particles can do so. As for light, QED (Quantum Electro-Dynamics) requires photons, which are quanta of light, to be massless and consequently light can propagate with the velocity of the upper limit C .³ Put differently, C is not in essence the velocity of light but light can move at the velocity C since its quanta 'photons' happen to be massless through the quantization of Maxwell's electrodynamics. Incidentally, gravitational waves in general relativity can propagate at the velocity C as well since their quanta 'gravitons' are massless.

In this way, by starting with both the principle of special relativity and the principle of a universal limiting speed without referring to the existence of light, one can reproduce the Lorentz transformation and the composition law of velocities in Einstein's special relativity. An important point in this approach is that the universal limiting speed C is to be determined only through experiment. It is this experiment that we are interested in connection with a resolution to the Cohen-Glashow effect of superluminal neutrinos.

Before delving into our resolution [13], let us review the Cohen-Glashow effect briefly [4]. The basic observation behind the Cohen-Glashow effect is that a neutrino moving faster than the velocity of light loses its kinetic energy by emitting something via weak interaction even if it does not possess electric charges. The most dominant decay process of a muon neutrino, which mainly constitutes the OPERA beam, is given by $\nu_\mu \rightarrow \nu_\mu + e^+ + e^-$. It turns out that after some calculations the terminal energy of a neutrino detected at Gran Sasso is about 12.5 GeV, so few neutrinos with energies in excess of 12.5 GeV reach the detector. Unfortunately, since the OPERA detector observes neutrinos with the mean energy of 17.5 GeV

³We identify the velocity of a photon with that of light.

ranging up to 50GeV, the Cohen-Glashow effect rules out an interpretation of the OPERA neutrinos being superluminal [4]. Furthermore, the ICARUS group also reaches the same conclusion as Cohen and Glashow by reanalyzing their data [8].

Now we are ready to present our resolution to the Cohen-Glashow effect. First, let us note that the Cohen-Glashow effect is a peculiar feature of superluminal particles. It is worthwhile to recall that special relativity never forbids existence of superluminal particles but only prohibits superluminal particles from becoming subluminal particles, and vice versa because the proper Lorentz transformation does not connect superluminal motion with subluminal one. Thus, if a particle is superluminal, we cannot take the rest frame for the particle since its minimum speed must be more than a universal limiting speed. On the other hand, if a particle is subluminal, by performing a suitable Lorentz transformation we can always take the rest frame for this particle, thereby kinematically forbidding the Cherenkov-like process of superluminal neutrinos to occur. Hence, our resolution to the Cohen-Glashow effect amounts to saying that the OPERA neutrinos are not superluminal but subluminal in order to avoid the Cohen-Glashow effect [13]. Then, a natural question arises whether our resolution is against the interpretation of superluminal neutrinos by OPERA group or not?

In order to answer this question, let us note that the causal structure, in other words, the fact whether a particle is superluminal, light-like or subluminal, is determined by the universal limiting speed C , which should be measured by experiment as mentioned above. There are different methods to determine the value of C experimentally. One natural way is to measure the actual speed of light in vacuum, which can be done in various astronomical and earth-based setups. Here we wish to insist that astronomical measurement should be done in outer space far from stars and planets in order to avoid the influence of dark matters. The experiments for measuring the speed of light in outer space have been thus far done by using the earth and various planets such as the sun and the moon in the solar system. But these experiments cannot help receiving the influence of dark matters since dark matters have a tendency to gather near massive objects such as stars and planets via a gravitational interaction. The universal limiting speed C at hand must be measured in a setup for which there is no coupling to dark matters.

As is well known, the phase velocity v at which light propagates in a medium such as water or air, is reduced by the refractive index n of the medium as $v = \frac{C}{n} < C$. If there are some undetectable media on the earth like dark matters, it is plausible that the observed speed c of light on the earth also becomes smaller than the universal limiting speed C , that is $c < C$. Then, it is reasonable to conjecture that the observed speed v_ν of the OPERA neutrinos is larger than the observed speed c of light on the earth while it is smaller than the universal limiting speed C ⁴

$$c < v_\nu < C. \quad (8)$$

With this conjecture, the OPERA neutrinos should be regarded as subluminal neutrinos since

⁴Of course, when there is no coupling between photons and dark matters, the observed speed of light coincides with the universal limiting speed C .

the causal structure of events are now defined with respect to the universal limiting speed C as mentioned before.

In other words, what we wish to present as a resolution to the Cohen-Glashow effect is the following: the OPERA neutrinos might be superluminal ($c < v_\nu$) if one compares the velocity of a neutrino with the observed velocity c of light on the earth, but since superluminality or subluminality must be determined on the basis of the universal limiting speed C , the OPERA neutrinos are actually subluminal ($v_\nu < C$) in comparison with the true velocity C .

3 Models with two velocities of light

Now let us note that with our conjecture mentioned above, the problem of the bremsstrahlung effect is converted to a different problem, which can be stated as follows: "Can we construct a physically plausible model which explains how a universal limiting speed C could be changed to the observed speed c of light on the earth by the influence of dark matters?"⁵

In our previous work [13]⁶, we have presented such a model with the observed velocity of light by using a symmetric tensor field of spin 2. In this section, we will construct three types of models with the observed velocity of light by using three kinds of bosonic degrees of freedom, which are scalar, vector and symmetric tensor fields. These bosonic fields are sourced by the earth and create a classical background, to which a gauge field describing photons couples. The photons then propagate through an effective metric and consequently the universal limiting speed C will be slightly reduced to become the observed speed of light c on the earth. Note that these bosonic fields could be regarded as parts of many candidates for dark matters and behave as if they were media such as water or air.

3.1 Scalar field

Let us start with the case of a scalar field whose effective Lagrangian density is of form:⁷

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2M_*^4}\partial^\nu\Pi\partial^\alpha\Pi F_{\mu\nu}F^\mu{}_\alpha - \frac{1}{2}\partial_\mu\Pi\partial^\mu\Pi - \frac{m^2}{2}\Pi^2 + \frac{4\pi}{M}\Pi T, \quad (9)$$

where M_* is a mass scale which controls the strength of a coupling between the scalar field Π and the abelian gauge field A_μ . And M is another mass scale setting the strength of a coupling between the scalar field and the trace of the energy-momentum tensor which describes an effective energy-momentum except the gauge field of the earth. The gauge field strength $F_{\mu\nu}$

⁵See Ref. [20] on the meaning of the speed of light and the varying speed of light theories.

⁶See also related works [21, 22, 23, 24, 25, 26, 27].

⁷We make use of a flat metric $\eta_{\mu\nu} = \text{diag}(-1, +1, +1, +1)$ for raising or lowering indices. Moreover, we adopt the Planck units $C = \hbar = G = 1$ by which all quantities become dimensionless multiples of the Planck length $L_{Pl} \equiv \left(\frac{G\hbar}{c^3}\right)^{\frac{1}{2}}$.

is defined by $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ as usual. Note that the second term is non-renormalizable so this Lagrangian density makes sense for energies less than M_* , beyond which the theory enters in the strong coupling region. Note that in the absence of the Π field, the photon travels at the universal limiting speed C since the gauge field satisfies the conventional Maxwell equations.

Now it is easy to rewrite (9) as

$$\mathcal{L} = -\frac{1}{4}(\eta^{\mu\alpha} - \frac{1}{M_*^4}\partial^\mu\Pi\partial^\alpha\Pi)(\eta^{\nu\beta} - \frac{1}{M_*^4}\partial^\nu\Pi\partial^\beta\Pi)F_{\mu\nu}F_{\alpha\beta} - \frac{1}{2}\partial_\mu\Pi\partial^\mu\Pi - \frac{m^2}{2}\Pi^2 + \frac{4\pi}{M}\Pi T, \quad (10)$$

thereby making it possible to read out an effective metric

$$g_{(A)}^{\mu\nu} = \eta^{\mu\nu} - \frac{1}{M_*^4}\partial^\mu\Pi\partial^\nu\Pi, \quad (11)$$

on which the photon propagates.

Next, let us derive the equation of motion for the scalar field Π

$$(\square - m^2)\Pi = -\frac{4\pi}{M}T + \frac{1}{M_*^4}\partial^\alpha(\partial^\nu\Pi F_{\mu\nu}F^\mu{}_\alpha). \quad (12)$$

For the range of energies and distances of our interests, the linearized analysis is fully sufficient and reliable, so we confine ourselves to the linearized equation of motion of Eq. (12)

$$(\square - m^2)\Pi = -\frac{4\pi}{M}T. \quad (13)$$

Here $T_{\mu\nu}$ is taken as a non-relativistic, static and spherically symmetric source of the earth's mass M_\oplus

$$T_{00} = M_\oplus\delta^3(r), \quad (14)$$

and the other components of $T_{\mu\nu}$ are vanishing. Then, the trace of $T_{\mu\nu}$ is given by

$$T \equiv \eta^{\mu\nu}T_{\mu\nu} = -T_{00} = -M_\oplus\delta^3(r). \quad (15)$$

Thus, for the static configuration, Eq. (13) reads

$$(\Delta - m^2)\Pi = 4\pi\frac{M_\oplus}{M}\delta^3(r), \quad (16)$$

so the solution takes the form

$$\Pi = -\frac{M_\oplus}{M}\frac{1}{r}e^{-mr}. \quad (17)$$

We postulate that the Compton wave-length of the Π field is the order of planetary distances, $\frac{1}{m} \gg r$, so we effectively set $m = 0$. Then, we obtain

$$\Pi = -\frac{M_\oplus}{M}\frac{1}{r}. \quad (18)$$

With this configuration, an effective space-time on which the photon propagates has the line element in the spherically symmetric coordinates

$$ds^2 \equiv g_{(A)\mu\nu} dx^\mu dx^\nu = -dt^2 + \frac{1}{1 - \frac{l^4}{r^4}} dr^2 + r^2 d\Omega_2^2, \quad (19)$$

where $d\Omega_2^2 \equiv d\theta^2 + \sin^2 \theta d\phi^2$ and we have defined $l \equiv \frac{1}{M_*} \sqrt{\frac{M_\Phi}{M}}$.

Whenever one obtains a prediction from general relativity the question always arises or should arise whether the result obtained really refers to an objective physical measurement or whether it has folded into arbitrary subjective elements dependent on our choice of coordinate system [28]. In the case at hand, one should ask oneself what the predictive change in the velocity of the photon really has to do with the positions of observed places. In fact, it has been already pointed out that the quantities defined in general relativity as an *average* velocity of massless particles traveling between two distant points can either sub- or superluminal depending on the position of the observer and the form of the trajectory in the gravitational field [29]. Thus, one has to consider what velocity is most suitable in the present physical setting.

In this article, we shall adopt a definition of an effective local velocity found by Einstein [30]. Usually, in calculating the precession of the perihelia of the mercury and the bending of light, the Schwarzschild solution to Einstein's equations is utilized, but Einstein himself has made use of the weak-field approximation of a gravitational field in the isotropic coordinates. Thus, following Einstein, we will look for the isotropic coordinates corresponding to the line element (19) as follows:

$$\begin{aligned} ds^2 &= -dt^2 + \frac{1}{1 - \frac{l^4}{r^4}} dr^2 + r^2 d\Omega_2^2 \\ &= -dt^2 + A(\bar{r})^2 (d\bar{r}^2 + \bar{r}^2 d\Omega_2^2) \\ &= -dt^2 + A(\bar{r})^2 (d\bar{x}^2 + d\bar{y}^2 + d\bar{z}^2), \end{aligned} \quad (20)$$

where $\bar{r}^2 = \bar{x}^2 + \bar{y}^2 + \bar{z}^2$. Then, it turns out that $A(\bar{r})$ is given by

$$A(\bar{r}) = \frac{r}{\bar{r}} = \frac{l}{\sqrt{1 + \sqrt{1 - \frac{l^4}{r^4}}}}. \quad (21)$$

Then, an effective velocity $c(r)$ of a photon is defined as

$$c(r) \equiv \sqrt{\frac{l^2}{2} \left[\left(\frac{d\bar{x}}{dt} \right)^2 + \left(\frac{d\bar{y}}{dt} \right)^2 + \left(\frac{d\bar{z}}{dt} \right)^2 \right]}, \quad (22)$$

where an overall factor $\frac{l^2}{2}$ is introduced for normalization. With this definition, the observed velocity of light in vacuum on the earth becomes

$$c(r) \equiv \frac{C}{\sqrt{2}} \left(1 + \sqrt{1 - \frac{l^4}{r^4}} \right)^{\frac{1}{2}}, \quad (23)$$

where for convenience the universal limiting velocity C is recovered. Note that at the spatial infinity $r \rightarrow \infty$, this observed velocity of light coincides with the universal limiting velocity C , that is,

$$\lim_{r \rightarrow \infty} c(r) = C. \quad (24)$$

Furthermore, note that the difference between C and $c(r)$ is positive-definite as required from our conjecture

$$C - c(r) = \frac{C}{8} \frac{l^4}{r^4} > 0, \quad (25)$$

which holds when $\frac{C}{8} \frac{l^4}{r^4} \ll 1$.

On the earth, the observed velocity of light is given by $c(r)$ when the photons are located at the place whose distance in the radial direction from the center of the earth is r . Since it is assumed that the speed of a neutrino, denoted as v_ν , takes a definite value and is smaller than the universal limiting speed C but larger than the observed speed $c(r)$

$$c(r) < v_\nu < C, \quad (26)$$

we have a superluminal neutrino for the velocity $c(r)$ as observed in the OPERA experiment

$$\beta(r) \equiv \frac{v_\nu - c(r)}{c(r)} > 0, \quad (27)$$

whereas we have a subluminal neutrino for the universal limiting speed C

$$\beta \equiv \frac{v_\nu - C}{C} < 0. \quad (28)$$

Recalling that the property of superluminality or subluminality of neutrinos is defined by using the universal limiting velocity C , the OPERA neutrinos are actually not superluminal but subluminal as emphasized before. Hence, we do not have Cherenkov-like radiation or the Cohen-Glashow effect for the OPERA neutrinos at all since we can always take the rest frame for the subluminal neutrino. This is our resolution to the problem of the bremsstrahlung effect of the OPERA neutrinos. It is worthwhile to notice that our solution is purely kinematical as desired.

Let us show that the results of OPERA and SN1987A give us useful information on two mass scales M_* and M . First, let us note that the OPERA result yields a condition for the dimensionless quantity $\beta(r)$

$$\beta(R_\oplus) \equiv \frac{v_\nu - c(R_\oplus)}{c(R_\oplus)} \approx 2.5 \times 10^{-5}, \quad (29)$$

where R_\oplus is the radius of the earth and takes the value $R_\oplus = 6.4 \times 10^8 \text{ cm}$.

Next, let us recall the fact that neutrinos from SN1987A to the earth travel at almost the same velocity as the velocity of light in vacuum, so we would have a relation

$$v_\nu \approx C. \quad (30)$$

With the help of this relation (30), Eq. (29) can be rewritten as

$$\beta(R_\oplus) \approx \frac{1}{8} \frac{l^4}{R_\oplus^4} = \frac{1}{8M_*^4} \left(\frac{M_\oplus}{M} \right)^2 \frac{1}{R_\oplus^4} \approx 2.5 \times 10^{-5}. \quad (31)$$

Introducing two length scales $L_* = \frac{1}{M_*}$ and $L = \frac{1}{M}$, we have an equation

$$L_*^4 L^2 \approx 8 \times 10^{-4} L_{Pl}^4 \frac{R_\oplus^4}{R_{\oplus SS}^2} \approx 10^{-99} (cm)^6, \quad (32)$$

where L_{Pl} , $R_{\oplus SS}$ are respectively the Planck length and the Schwarzschild radius of the earth, and are explicitly given by $L_{Pl} = 1.6 \times 10^{-33} cm$, $R_{\oplus SS} = 0.89 cm$.

For instance, if $L_* \approx L$, Eq. (32) gives us

$$L_* \approx L \approx 10^{-17} cm, \quad M_* \approx M \approx 1 TeV. \quad (33)$$

It is of interest that in this instance the mass scale M_* is approximately beyond the energy scale of Standard Model. Recall that in the energy region above M_* , the coupling between the gauge field and the scalar field becomes so strong that our effective theory is expected to be replaced to a unknown, sensible and renormalizable UV-completed theory.

At this stage, it is tempting to identify the Π scalar field with part of many candidates for dark matters. Actually, this identification seems to be consistent with the results of OPERA and SN1987A at the same time by the following reasoning: In general, dark matters are expected to be trapped in the vicinity of massive objects such as stars and the earth compared to empty regions of outer space owing to a gravitational interaction. The reason why neutrinos from SN1987A to the earth traveled at almost the same velocity as the velocity of light in vacuum is that since there are not enough dark matters in outer space, the neutrinos from SN1987A propagated at the universal limiting speed C without interacting with dark matters. On the other hand, since it is expected that there are sufficient dark matters on the earth, the interaction between the photons and dark matters reduces the speed of light on the earth to the smaller observed velocity $c(R_\oplus)$ from the larger universal limiting speed C . Accordingly, by the difference of existence of dark matters, the OPERA neutrinos travel on the earth at the speed less than the universal limiting speed C while the SN1987A ones propagate in the interstellar space at the velocity almost equal to the universal limiting speed C .

3.2 Vector field

Now we wish to consider the case of a new gauge field denoted as B_μ , which should not be confused with the gauge field A_μ describing the conventional photons. A line of argument in this subsection is similar to the case of the scalar field Π treated in the previous subsection.

The starting Lagrangian density is defined as

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2M_*^4}G^{\nu\alpha}G^{\beta}_{\alpha}F_{\mu\nu}F^{\mu}_{\beta} - \frac{1}{4M_*^8}G^{\mu\rho}G^{\alpha}_{\rho}G^{\nu\sigma}G^{\beta}_{\sigma}F_{\mu\nu}F_{\alpha\beta} \\
&- \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - B_{\mu}J^{\mu} \\
&= -\frac{1}{4}(\eta^{\mu\alpha} - \frac{1}{M_*^4}G^{\mu\rho}G^{\alpha}_{\rho})(\eta^{\nu\beta} - \frac{1}{M_*^4}G^{\nu\sigma}G^{\beta}_{\sigma})F_{\mu\nu}F_{\alpha\beta} \\
&- \frac{1}{4}G_{\mu\nu}G^{\mu\nu} - B_{\mu}J^{\mu},
\end{aligned} \tag{34}$$

where the field strength $G_{\mu\nu}$ of the new gauge field B_{μ} is defined as $G_{\mu\nu} = \partial_{\mu}B_{\nu} - \partial_{\nu}B_{\mu}$, and B_{μ} couples to the electro-magnetic current J^{μ} of the earth. The second expression in the Lagrangian density (34) urges us to take an effective metric

$$g_{(A)}^{\mu\nu} = \eta^{\mu\nu} - \frac{1}{M_*^4}G^{\mu\rho}G^{\nu}_{\rho}, \tag{35}$$

on which the photon propagates.

Taking variation of the scalar field B_{μ} yields the equation of motion

$$\begin{aligned}
\partial_{\mu}G^{\mu\nu} &= J^{\nu} - \frac{1}{M_*^4}\partial_{\alpha}[(G^{\beta\alpha}F_{\mu}^{\nu} - G^{\beta\nu}F_{\mu}^{\alpha})F^{\mu}_{\beta}] \\
&+ \frac{1}{M_*^8}\partial_{\alpha}[(G^{\mu\alpha}F^{\nu}_{\rho} - G^{\mu\nu}F^{\alpha}_{\rho})G^{\rho\sigma}G^{\beta}_{\sigma}F_{\mu\beta}].
\end{aligned} \tag{36}$$

In the linearized level, Eq. (36) reduces to the form

$$\square B_{\mu} = J_{\mu}, \tag{37}$$

where we have chosen the Lorentz gauge $\partial_{\mu}B^{\mu} = 0$.

Now we assume that the electromagnetic current has a static, spherically symmetric magnetic source

$$J_{\theta} = 4\pi\alpha\delta^3(r), \tag{38}$$

where α is the magnetic dipole moment of the earth. Then, we obtain the solution to the linearized equation of motion (37)

$$B_{\theta} = -\frac{\alpha}{r}, \tag{39}$$

from which we have the non-vanishing magnetic field

$$G_{r\theta} = \frac{\alpha}{r^2}. \tag{40}$$

As in the scalar field, it is straightforward to derive an effective metric by using Eq's. (35) and (40). Then, the line element reads

$$ds^2 = -dt^2 + \frac{1}{1 - \frac{1}{M_*^4} \frac{\alpha^2}{r^6}} (dr^2 + r^2 d\theta^2) + r^2 \sin^2 \theta d\varphi^2. \quad (41)$$

Using the spherical symmetry, we take $\varphi = 0$. Then, we have

$$\begin{aligned} ds^2|_{\varphi=0} &= -dt^2 + \frac{1}{1 - \frac{1}{M_*^4} \frac{\alpha^2}{r^6}} (dr^2 + r^2 d\theta^2) \\ &= -dt^2 + \frac{1}{1 - \frac{1}{M_*^4} \frac{\alpha^2}{r^6}} (dx^2 + dz^2), \end{aligned} \quad (42)$$

from which we can read out an effective local velocity of a photon

$$\begin{aligned} c(r) &\equiv C \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \\ &= C \sqrt{1 - \frac{1}{M_*^4} \frac{\alpha^2}{r^6}}. \end{aligned} \quad (43)$$

This observed speed of light also satisfies the relation (24), and the difference between the universal limiting speed C and the observed one $c(r)$ becomes positive-definite as in the scalar field.

From the results of OPERA and SN1987A, the dimensionless quantity β must take the following value

$$\beta(R_{\oplus}) \equiv \frac{v_{\nu} - c(R_{\oplus})}{c(R_{\oplus})} \approx \frac{1}{2M_*^4} \frac{\alpha^2}{R_{\oplus}^6} \approx 2.5 \times 10^{-5}. \quad (44)$$

Moreover, since the magnetic field on the earth is about 0.5×10^{-4} Tesla, we have a relation

$$\frac{\alpha}{R_{\oplus}^3} = 0.5 \times 10^{-4}. \quad (45)$$

Recovering dimensional factors, it turns out that the mass scale is described as

$$M_*^4 = 0.2 \times 10^5 \frac{\hbar^3}{\mu_0 c^5} \left(\frac{\alpha}{R_{\oplus}^3} \right)^2, \quad (46)$$

where μ_0 denotes magnetic permeability of the vacuum on the earth. This expression gives us the mass scale in this theory

$$M_* \approx 1eV. \quad (47)$$

Such a low mass scale has also appeared in our previous work [26] where a superluminal neutrino has been obtained by coupling the neutrino field to a new gauge field. However, a physical interpretation of the two low mass scales might be different since in the present model there are two gauge fields B_{μ} and A_{μ} whereas in the previous model there is only one gauge field A_{μ} which could be identified with the conventional photon field.

3.3 Tensor field

A symmetric tensor field was originally dealt with in Ref. [21] in order to make neutrinos superluminal to explain the OPERA results. In this subsection, we make use of the symmetric tensor field to obtain the observed speed of light smaller than the universal limiting speed. We will again follow a similar path of thought to the cases of scalar and vector fields.

The Lagrangian density consists of three parts which are part of the gauge field and its coupling with the tensor, that of massive gravity of Fierz-Pauli type, and that of source as follows:

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2M_*}h^{\nu\alpha}F_{\mu\nu}F^\mu{}_\alpha - \frac{1}{4M_*^2}h^{\mu\alpha}h^{\nu\beta}F_{\mu\nu}F_{\alpha\beta} \\
&+ \frac{1}{2}h_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\beta}h_{\alpha\beta} - \frac{m^2}{2}(h_{\mu\nu}^2 - h^2) + \frac{1}{M}h_{\mu\nu}T^{\mu\nu} \\
&= -\frac{1}{4}(\eta^{\mu\alpha} - \frac{1}{M_*}h^{\mu\alpha})(\eta^{\nu\beta} - \frac{1}{M_*}h^{\nu\beta})F_{\mu\nu}F_{\alpha\beta} \\
&+ \frac{1}{2}h_{\mu\nu}\mathcal{E}^{\mu\nu\alpha\beta}h_{\alpha\beta} - \frac{m^2}{2}(h_{\mu\nu}^2 - h^2) + \frac{1}{M}h_{\mu\nu}T^{\mu\nu},
\end{aligned} \tag{48}$$

where $h_{\mu\nu}$ is defined as $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$, and $\mathcal{E}^{\mu\nu\alpha\beta}$ is an operator stemming from the Einstein-Hilbert term which is defined for a general symmetric tensor $Z_{\alpha\beta} = Z_{\beta\alpha}$ as

$$\mathcal{E}^{\mu\nu\alpha\beta}Z_{\alpha\beta} = \square Z^{\mu\nu} - \eta^{\mu\nu}\square Z - \partial^\mu\partial_\alpha Z^{\nu\alpha} - \partial^\nu\partial_\alpha Z^{\mu\alpha} + \partial^\mu\partial^\nu Z + \eta^{\mu\nu}\partial_\alpha\partial_\beta Z^{\alpha\beta}. \tag{49}$$

The second expression in the Lagrangian density (48) leads to an effective metric

$$g_{(A)}^{\mu\nu} = \eta^{\mu\nu} - \frac{1}{M_*}h^{\mu\nu}. \tag{50}$$

In contrast to the scalar and vector cases with a factor $\frac{1}{M_*^4}$, this metric depends on $\frac{1}{M_*}$, which makes the mass scale larger around the Planck scale as seen shortly.

The equation of motion for the tensor field $h_{\mu\nu}$ is of form

$$\mathcal{E}_{\mu\nu}{}^{\alpha\beta}h_{\alpha\beta} - m^2(h_{\mu\nu} - \eta_{\mu\nu}h) = -\frac{1}{M}T_{\mu\nu} + \frac{1}{2M_*}(\eta^{\alpha\beta} - \frac{1}{M_*}h^{\alpha\beta})F_{\mu\alpha}F_{\nu\beta}. \tag{51}$$

In the linearized level, Eq. (51) becomes the form

$$\mathcal{E}_{\mu\nu}{}^{\alpha\beta}h_{\alpha\beta} - m^2(h_{\mu\nu} - \eta_{\mu\nu}h) = -\frac{1}{M}T_{\mu\nu}. \tag{52}$$

Taking the trace of Eq. (52), one obtains

$$\square h - \partial_\alpha\partial_\beta h^{\alpha\beta} = \frac{3}{2}m^2h + \frac{1}{2M}T. \tag{53}$$

Moreover, taking the divergence of Eq. (52) produces an equation

$$\partial^\mu h_{\mu\nu} - \partial_\nu h = 0. \tag{54}$$

Together with Eq. (53) and Eq. (54), we can derive an equation

$$h = -\frac{1}{3m^2 M} T. \quad (55)$$

Then, Eq. (52) with the help of Eq's. (54) and (55) can be cast to the form

$$(\square - m^2)h_{\mu\nu} = -\frac{1}{M} \left[T_{\mu\nu} - \frac{1}{3}(\eta_{\mu\nu} - \frac{1}{m^2} \partial_\mu \partial_\nu) T \right]. \quad (56)$$

Since we take a non-relativistic, static and spherically symmetric source (14) and the long Compton wave-length approximation as in the scalar field, we finally obtain the equation of motion for $h_{\mu\nu}$

$$\Delta h_{00} = -\frac{2}{3M} M_\oplus \delta^3(r), \quad \Delta h_{ij} = -\frac{1}{3M} M_\oplus \delta^3(r) \delta_{ij}, \quad (57)$$

where indices i, j indicate spatial ones $i = 1, 2, 3$ and the contribution proportional to total derivative is neglected owing to conservation of the electro-magnetic current. Using the Poisson equation $\Delta \frac{1}{r} = -4\pi \delta^3(r)$, Eq. (57) can be solved to the form

$$h_{00} = \frac{1}{6\pi} \frac{M_\oplus}{M} \frac{1}{r}, \quad h_{ij} = \frac{1}{12\pi} \frac{M_\oplus}{M} \frac{1}{r} \delta_{ij}. \quad (58)$$

As before, it is easy to calculate the effective metric (50) by using Eq. (58). Then, taking the inverse of the effective metric, the line element is found to be

$$ds^2 = - \left(1 - \frac{1}{6\pi} \frac{M_\oplus}{M_* M} \frac{1}{r} \right) dt^2 + \left(1 + \frac{1}{12\pi} \frac{M_\oplus}{M_* M} \frac{1}{r} \right) \delta_{ij} dx^i dx^j. \quad (59)$$

Since the trajectory of light is null, $ds^2 = 0$, the effective local velocity of a photon reads

$$\begin{aligned} c(r) &\equiv C \sqrt{\delta_{ij} \frac{dx^i}{dt} \frac{dx^j}{dt}} \\ &= C \sqrt{\frac{1 - \frac{1}{6\pi} \frac{M_\oplus}{M_* M} \frac{1}{r}}{1 + \frac{1}{12\pi} \frac{M_\oplus}{M_* M} \frac{1}{r}}} \\ &\approx C \left(1 - \frac{1}{8\pi} \frac{M_\oplus}{M_* M} \frac{1}{r} \right). \end{aligned} \quad (60)$$

This observed speed of light also satisfies the relation (24), and the difference between C and $c(r)$ becomes positive-definite as in the both scalar and vector fields.

Again the results of OPERA and SN1987A require the dimensionless quantity β to take the following value

$$\beta(R_\oplus) \equiv \frac{v_\nu - c(R_\oplus)}{c(R_\oplus)} \approx \frac{1}{8\pi} \frac{M_\oplus}{M_* M} \frac{1}{R_\oplus} \approx 2.5 \times 10^{-5}. \quad (61)$$

Using this expression and recovering dimensional factors, it turns out that the two mass scales are constrained to be

$$M_* M = \frac{1}{40\pi} \times 10^5 M_{Pl}^2 \frac{R_{\oplus SS}}{R_{\oplus}} \approx 10^{-6} M_{Pl}^2. \quad (62)$$

This result is very similar to that obtained by Dvali and Vikman in Ref. [21] so that we think that the present model passes various phenomenological constraints investigated in [21, 22]. For example, the absence of any observable long-range fifth force of gravity-type and the maximal violation of equivalence principle imply $M \approx 10^2 M_{Pl}$ and $M_* \approx 10^{-8} M_{Pl}$. The latter mass scale is also consistent with the correction to the cooling rate of stars because of production of $h_{\mu\nu}$. Finally, we remark that in contrast to the theory in [21], in our theory there is no sign asymmetry of the couplings so our theory could have a consistent UV-completion.

The last, but not least, let us comment on the stability of a photon which is known to be stable [3]. In this article, we have introduced specific non-renormalizable interactions to obtain the observed speed of light from the universal limiting speed. These interaction terms have a possibility of triggering the spontaneous decay of a photon, thus losing its stability. However, it turns out that the decay process occurs at the loop levels (in cases of the both scalar and tensor models, it occurs from the two loop level while in the vector model, at the one loop level), and it is greatly suppressed by the mass scale M_* . For instance, since in the scalar model, the amplitude $\gamma + \gamma \rightarrow \Pi + \Pi$ is schematically described as $\left(\frac{1}{M_*^4}\right)^3 \left(\int^\Lambda d^4 p \frac{p^6}{p^4}\right)^2 \approx \left(\frac{\Lambda}{M_*}\right)^{12}$, the amplitude is so tiny as long as $\Lambda \ll M_*$, thereby ensuring the stability of a photon in this model. In a similar way, it is easily shown that a photon is stable for $\Lambda \ll M_*$ in all the models.

4 Discussion

In this article, we have explained our resolution to the Cohen-Glashow effect associated with the OPERA superluminal neutrinos in detail. After all, our resolution is equivalent to saying that the OPERA neutrinos are not actually superluminal but subluminal since they travel at the speed slower than a universal limiting speed.

Relating to our resolution, we have also pointed out that the principle of invariant speed of light in special relativity can be replaced with the principle of a universal limiting speed. This universal limiting speed is the maximum velocity of elementary particles and information, and the observed speed of light must be equal to or smaller than the universal limiting speed where the equality holds when the coupling of a photon with dark matters is switched off. However, since our earth is surrounded by a cloud of dark matters and cannot escape from the influence, the observed speed of light must be always less than the universal limiting speed. As emphasized in this article, the causal structure of all the events should be defined through not the observed speed of light on the earth but the universal limiting speed. Furthermore, this universal limiting speed is nothing but the velocity appearing in various formulae of special relativity and electrodynamics such as the Lorentz transformation.

Our resolution to the Cohen-Glashow effect is on the kinematical grounds and thus strictly forbids the OPERA neutrinos to emit a pair of electron and positron via the bremsstrahlung effect. It is then natural to ask ourselves if our resolution also provides a resolution to the other problems associated with the OPERA results. In particular, it is now known that the other challenging theoretical issue lies in pion decay process [5, 6, 7] where it was found that the decay of charged pion $\pi^+ \rightarrow \mu^+ + \nu_\mu$, which is nothing but the neutrino production process in the OPERA experiment, becomes kinematically forbidden for $E_\nu > 5\text{GeV}$, which is obviously inconsistent with the OPERA results. It is worth stressing that our resolution also provides a solution to this problem since this decay process is automatically prohibited whenever the OPERA neutrinos are subluminal.

Nevertheless, there remain some issues to be clarified in future. In particular, we should investigate various phenomenological implications of the present models in more detail. For instance, the couplings between the new bosonic fields and photons would yield observable effects on the bending and red-shift of light. Furthermore, we should look for astrophysical and cosmological constraints coming from CMB and supernova Ia. Dark matters in the universe are known to be so dilute but the propagation over cosmological distances might give rise to an accumulated contribution even for a tiny coupling. We wish to take into consideration these problems in the next publication.

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